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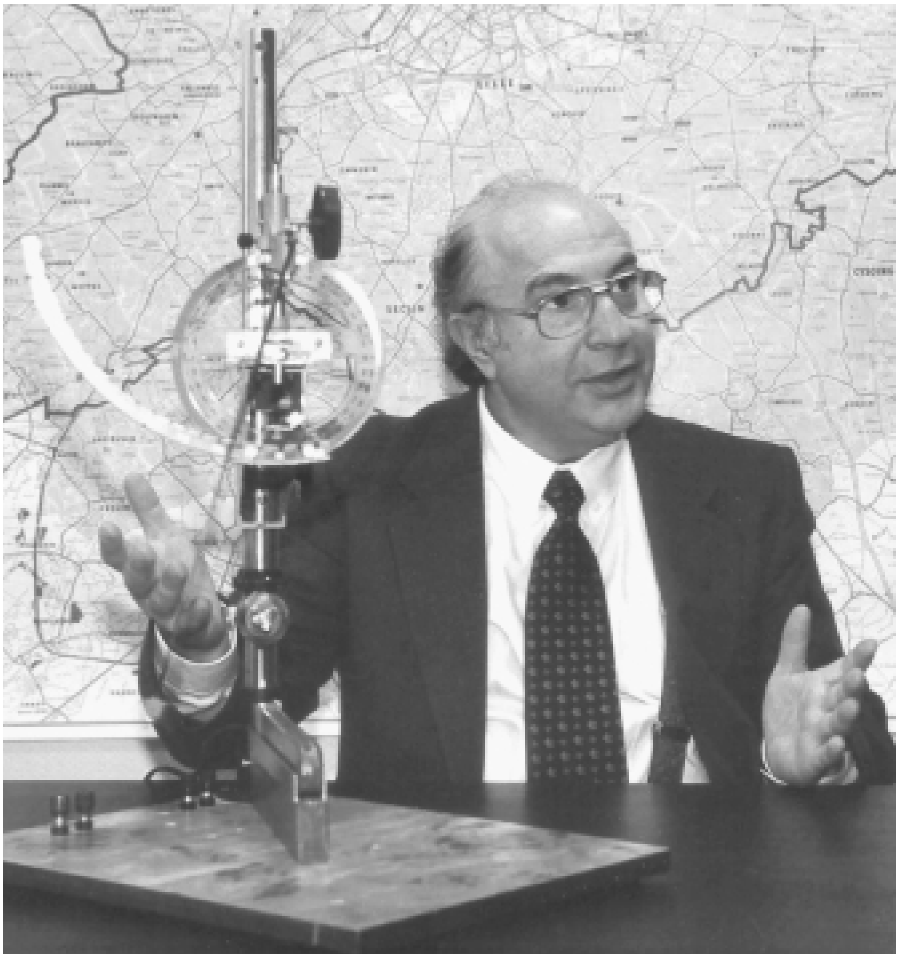
Amplitude Quantization As an Elementary Property of Macroscopic Vibrating Systems

by Jonathan Tennenbaum



Fusion

Danil Doubochinski with his pendulum.

The discovery of a new physical principle, argumental oscillations, pokes holes in textbook physics and emphasizes the need to break out of the shackles of formal mathematics and of Newtonianism in general.

One of the strong points of physics and engineering in the Soviet Union was the study of what are sometimes called "nonlinear oscillations." A great number of important experimental and theoretical results emerged, of which only a part became generally known in the West.

Virtually unnoticed in the West, was the discovery of the phenomenon of quantization of amplitudes in certain macroscopic oscillating systems. This phenomenon, and the principle behind it, were originally discovered in 1968-1969 by Danil Doubochinski and his brother Yakov, while students at Moscow University. Subsequently, a number of leading laboratories in the Soviet Union carried out extensive theoretical and experimental investigations of the phenomenon, establishing the existence of a new class of vibratory processes—so-called "argumental oscillations"—and of a new technological principle, having an enormous scope of potential applications. Most of them derive from the ability of argumental oscillations, to efficiently couple together oscillational processes at frequencies differing by two or more orders of magnitude. For a variety of reasons, however, only a very few applications were brought to full fruition in the Soviet period; and those that were, remained in a limited domain that escaped broad notice in the international community.

In the meantime, Danil Doubochinski, now working in Paris, has called attention to the fundamental importance of his discovery as a kind of bridge between so-called classical and quantum physics. It provides, in his view, an answer to a central question which Planck, Einstein, Schrödinger and others had posed at the beginning of the 20th Century, but were unable to answer in a satisfactory way: the question of the physical origin and nature of the apparent discontinuities—the so-called "quantum jumps"—in the interaction between atoms of matter and the electromagnetic field.

At the same time, Danil Doubochinski and a group of collaborators in France have succeeded in developing several specific technologies, based on the principle of argumental oscillations, to the point of ripeness for commercial application. These include an extremely efficient means for the atomization and vaporization of liquids by means of "resonant cavitation" and, on that basis, a revolutionary new technology for industrial refrigeration, having enormous potential economic benefits. Other near-term applications include low-cost production of drinking water and a highly efficient vibratory process for the preparation of emulsions. Beyond this, there remains a broad field of potential applications to such areas as the generation and transmission of electrical power, electrical motors and propulsion systems of a new type, radio frequency and microwave technologies, vibrational methods for processing of liquid and solid materials, new approaches to nuclear fusion, and more.

The author had the occasion to meet several times with Danil Doubochinski, to discuss his work and to witness some very beautiful experimental demonstrations of the quantization effect. One of them—a pendulum interacting with an alternating magnetic field—is so simple, that it belongs in every high school physics classroom.

An initial report on Doubochinski's work was published in March 2001 in the French-language magazine *Fusion.* The subsequent technological developments, and a growing appreciation of the pedagogical and scientific value of the work, justify the publication here of a revised form of the original report, including a more adequate discussion of argumental oscillations and the genesis of the discovery. We plan to follow this soon with a second article, covering some of the ongoing work on technological applications.

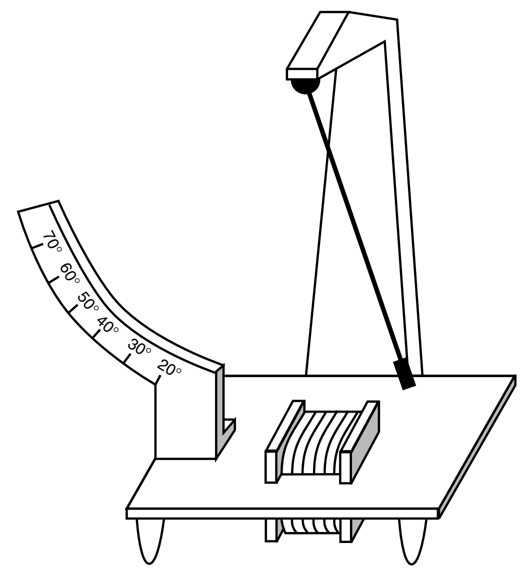


Figure 1
SCHEMATIC OF THE DOUBOCHINSKI PENDULUM

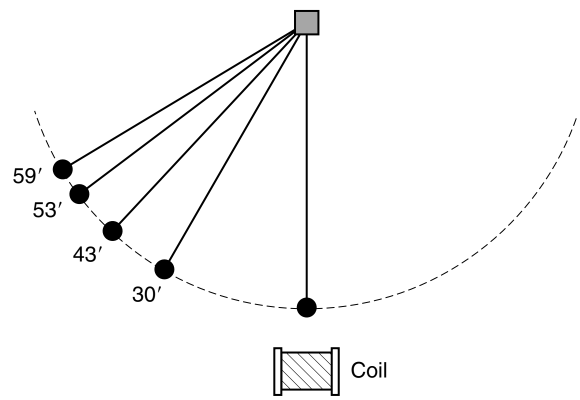


Figure 2
FIRST FOUR QUANTIZED AMPLITUDES
The four first quantized amplitudes of the Doubochinski pendulum, with a magnetic field frequency F *= 50 Hz.*

Doubochinski's Pendulum

The pendulum (Figure 1) consists of a rigid arm on a low-friction pivot, constrained thereby to move in a horizontal plane, with a small permanent magnet fixed at its free end. An electromagnet is installed just under the lowest point of the pendulum's motion—the vertical position of the pendulum—with its axis aligned horizontally in the plane of the pendulum's motion, in such a way, that at any moment the electromagnet exerts an accelerating or decelerating action on the permanent magnet at the end of the pendulum, depending on the polarity of the current supplied to the electromagnet and on the direction of the pendulum's motion. The axis-length of the electromagnet is chosen short, relative to the length of the pendulum, so that the action of the electromagnet on the pendulum becomes significant only over a small portion of the pendulum's motion, when the end of the pendulum is located within a relatively narrow zone of interaction corresponding approximately to the length of the electromagnet. This spatial inhomogeneity of the field, acting upon the pendulum, plays a key role in the genesis of amplitude quantization.

Now we connect the electromagnet with a source of sinusoidal alternating current, whose frequency *f* and voltage can be varied over a wide range (typically f = 20 to 3,000 Hz, for a pendulum with a natural period of about 0.5 sec). As soon as the current is sufficient for a significant interaction between the electromagnet and the pendulum to occur, we observe the following characteristic phenomena:

When released at any arbitrary starting position, the pendulum's motion evolves into one of a discrete set of stable oscillation modes, having sharply differing amplitudes, but approximately the same period of oscillation—close to the pendulum's undisturbed period (Figure 2). In each such mode, the energy lost by friction in the pendulum's motion, is compensated by net power transfer from the oscillating magnetic field, in a self-regulating manner. The system's "choice" among the discrete set of stable modes, is determined by the initial conditions.

Doubochinski's pendulum has the further remarkable property, shared by argumental oscillations in general, that the "quantized" amplitudes, and the corresponding stable modes of the pendulum, do not change appreciably, when the strength of the "external force" (the alternating field, in this case) is varied over a wide range. The amplitudes are highly sensitive, however, to changes in the frequency of the applied forces. The higher the applied frequency, the larger the array of stable modes that become accessible to the pendulum. (See Table 1).

Exactly this sort of behavior—strikingly different from that displayed by the linear resonators of classical mechanics—is characteristic of quantum processes in the microscopic domain, as exemplified by the photoelectric effect and the absorption of electromagnetic radiation by atoms and molecules.[1]

The pendulum's quantized modes are remarkably stable with respect to vibration and changes in the system's frictional and other parameters; large disturbances can cause the pendulum to "jump" from one mode to another, imitating the "quantum jumps" of atomic physics.

The effect does not depend on any special details of design or on the specific materials used in the construction of the electromagnet and pendulum. The system just described, in fact, merely exemplifies an entire class of macroscopic oscillating systems exhibiting similar "quantized" behavior. Some of these are more difficult to realize technically, but are more natural, from a theoretical physics point of view. One of them is extremely close to the theoretically idealized case of "elementary oscillators" interacting with an electromagnetic field, used by Max Planck in his investigation of the law of blackbody radiation. One suspects that the historical development of quantum physics would have taken a different course, had Planck and his contemporaries been familiar with the sort of phenomena, demonstrated by Doubochinski's pendulum.

It is remarkable, that despite the considerable academic and public attention paid in recent decades to so-called "nonlinear dynamics," "self-organization," "chaos theory," "synergetics," "dissipative systems," and so forth, no one seems to have pointed out an example so simple, so elementary, and at the same time so fundamental, as that discovered by Doubochinski. This embarrassing circumstance is no doubt due to the fact, that the bulk of research and publication activity on "nonlinearity," has had more to do with mathematicians' games, than with the mastery of physical reality. A more profound reason, we consider, is a lack of comprehension of the true, ontological meaning of nonlinearity. A truly nonlinear process is one, that by its very nature cannot be represented in a consistent and comprehensive manner by formal-mathematical methods.

Beyond 'Classical Mechanics'

At first glance, the processes studied by Doubochinski would appear to fall entirely inside the domain of classical (macroscopic) mechanics. Examining Doubochinski's pendulum, for example, any trained physicist or engineer can easily

F (Hz)	Pendulum amplitudes							
5					68°			
20	30°			59°		74°		85°
50	30°	43°	53°	59°	68°	74°	80°	85°

Table 1
THE STABLE QUANTIZED AMPLITUDES OF THE DOUBOCHINSKI PENDULUM AS A FUNCTION OF THE FREQUENCY OF THE MAGNETIC FIELD
The experimentally observed stable frequencies do not depend significantly upon the dimensions of the interaction zone (size of the coil), as long as the width of the zone remains small relative to the length of the pendulum.

write down a rather simple differential equation to describe its motion, applying the standard Lagrangian method for an appropriate choice of mathematical function describing the external force acting on the pendulum as a function of the time and space coordinates (See box, p. 54). For this reason, some might dismiss Doubochinski's work as a mere exercise, having no fundamental interest.

The situation is rather more subtle than it appears, however.

First, from a purely technical standpoint, our physicist or engineer will note that the differential equation, describing Doubochinski's pendulum according to classical mechanics, is of a type that cannot be solved, in explicit form, by any of the presently known methods of mathematical analysis. Furthermore, the quantized amplitudes, observed in actual physical experiments, do not manifest themselves in the usual sorts of computer-based numerical-approximation solutions (simulations) of the differential equation.[2]

Second, apart from the mathematical difficulties it introduces, the spatial inhomogeneity of the force-field in Doubochinski's pendulum (and systems of a similar type) means that the external force, experienced by the moving pendulum at any moment, depends not only on the time, but also on the momentary position of the pendulum itself. This dependence of the external force on position, which is notably absent in classical textbook discussions of so-called "forced oscillations," permits the pendulum, in a certain sense, to self-regulate its exchange of energy with the external source. This condition is key to the quantized behavior, demonstrated by Doubochinski's pendulum. He employs the technical term "argumental oscillations" to describe the general case, in which the momentary position or configuration of an oscillating system, enters as a variable into the functional expression for the external, oscillating force acting upon it. The possibility of self-regulation of energy-exchange is a general characteristic of argumental oscillations.

Third: Although Danil Doubochinski and his collaborators have developed mathematical methods for the analysis of amplitude quantization and other properties of argumental oscillations, those who are looking for a mathematical deduction of the phenomena from the "laws of classical mechanics," will be frustrated. Doubochinski's theoretical-mathematical analysis lacks the quality of logical completeness, which typifies the treatment of linear oscillators, for example, in textbooks of analytical mechanics. Accordingly, some critics regard his analysis of amplitude quantization as untrustworthy and even erroneous.

In fact, if we did not know, by direct experimental demonstration, that the phenomenon of amplitude quantification actually exists, then we would probably not be convinced by the analytical arguments that Doubochinski et al. have put forward on this account. But those arguments—which we shall briefly examine later in this article—were never intended to be a self-contained, aprioristic mathematical theory. They reflect on years of experimental investigations of real-life oscillating systems, and are intended to supplement—but not to replace!—those experimental results.

Far from claiming to deduce the behavior of his pendulum from "the laws of classical physics," Doubochinski sees in this behavior the manifestation of a new physical principle, which is not incorporated in classical physics as commonly understood. This point has given rise to considerable confusion, and necessitates a brief excursion into the issue of methodology, before we take a closer look at argumental oscillations.

Kepler vs. Lagrangian 'Virtual Reality'

Over the last 200 years, the influence of Lagrange's *Mechanique analytique* on prevailing modes of scientific education, has given rise to the widespread presumption, that so-called "classical mechanics" constitutes the perfect exemplar of a completed physical theory. It is presumed, that from the standpoint of physical principle, nothing fundamentally new could remain to be discovered in that domain. Danil Doubochinski disagrees.

Strictly speaking, of course, Planck's discovery of the quantum of action, and the subsequent elaboration of the so-called wave mechanics by Schrödinger, already imply a fundamental correction of classical mechanics. The standard textbook accounts assure us, however, that this correction, while significant in the domain of microscopic physical objects, can be virtually neglected when dealing with systems of macroscopic bodies. The reason given for this, is the practically infinitesimally small value of Planck's quantum, compared to the magnitudes of action involved in the motion of macroscopic bodies. The latter would include Doubochinski's pendulum and all other macroscopic systems belonging to the traditional domain of classical mechanics.

Physicists and engineers, who for generations have been drilled in the mathematical formalisms of Lagrange and Hamilton, often regard it as self-evident, that a macroscopic mechanical system is in principle fully equivalent to the corresponding set of differential or integral equations derived according to the Lagrangian or Hamiltonian methods of analytical mechanics. Many would hasten to add, of course, that in practice certain idealizations, simplifications, and approximations are always introduced, in order to make the mathematical equations more manageable. But this practice is purely pragmatic, and does not contradict the assumed, principled equivalence between the physical and mathematical systems.

Recent times have seen this view carried to the extreme, as some people have suggested that physics as a whole is already practically complete in terms of its foundations. The "fundamental forces" being essentially already known, all that remains is to solve the equations! This view has already found its expression in the growing tendency, in the teaching of physics and even in experimental physics itself, to replace actual experiments by computer-based "virtual experiments." The next step might be "virtual laboratories" staffed with "virtual scientists"!

However, the closely related trend toward use of large-scale computer simulations, to replace the costly and time-consuming practice of building and testing actual prototype systems, has led to some rather unpleasant consequences. The dangerous dynamic instability of Mercedes-Benz's famous computer-designed and computer-tested "A-Class" automobile, was revealed in 1997, when it repeatedly tipped over during independent driving tests, conducted after the car had already gone into production. Similarly, during the late-1990s the United States suffered a long series of catastrophic failures in the launches of computer-tested rocket systems, plus the total failure of two NASA Mars missions, which had functioned well in virtual reality simulations. Many more examples could be given.

The disasters caused by overreliance on computer simulations come as no big surprise to old timers in industrial science and engineering—people who know, from long and sometimes painful experience, the difference between the real-life behavior of physical systems and the virtual reality of textbook analytical mechanics.

The problem is not simply one of numerical accuracy, but a qualitative one: The mathematical methods of physics, while useful and indispensable in the hands of an experienced physicist or engineer, are by their very nature incapable of representing physical reality *per se*. The successful practice of technology always depends on the unique powers of the human mind, to conceptualize a physical process as a whole in terms of its underlying principles, and to correct for the errors that would inevitably flow from any blind use of formal mathematical and related methods. These are the same creative powers, which permit original scientists to uncover anomalies in areas thought to be completely understood by generally accepted scientific theory, and to discover new physical principles, not accounted for by existing, formal scientific knowledge.

Exactly this point was at the center of a very relevant struggle over the future of science in France two centuries ago, between the Republican circles associated with Monge, Carnot, Ampère, Arago, and Fresnel on the one side, and the oligarchic forces represented by Laplace, Cauchy, and others on the other. Unfortunately, Laplace and his backers were largely successful in their campaign to replace the original emphasis on physical geometry in the curriculum of the famous Ecole Polytechnique, by a curriculum centered on analytical mechanics in its most abstract form, including especially the Newtonian-Laplacian "celestial mechanics."

The politically backed imposition of Laplace's celestial mechanics as the supposed standard for mathematical physics, had nothing to do with its scientific merits. On the contrary: The utter failure of celestial mechanics to account for the most crucial feature of our solar system—the quantization of the planetary orbits according to harmonic principles, demonstrated by Johannes Kepler two centuries before—shows that the Newton-Laplace form of mathematical physics is intrinsically flawed and does not correspond to reality.

Here we meet Danil Doubochinski again. He sees the quantization of orbits in the solar system, as an astronomical manifestation of the same amplitude quantization principle, demonstrated on the laboratory scale by his pendulum and related electromechanical devices. Doubochinski himself has made a preliminary attempt to account for the values of the planetary orbits on the hypothesis that they represent a form of argumental oscillations.[3]

Freeing the Mind from the Slavery of Newtonianism

More significant, for our present purposes, is the pedagogical value of Doubochinski's pendulum, not least of all in connection with a critique of the Newtonian conception of force, which has sown deep-seated prejudices not only in the minds of physicists, but within culture as a whole. Danil Doubochinski himself justifiably blames Newton, and later Lagrange, for having introduced a fundamental fallacy into physics, relative to the original, far superior standpoint of Kepler. This involves at least three, subsumed conceptual flaws:

First, the present-day hegemonic conception of force, going back to Newton et al., implies the idea of a rigid, "slave-like" obeisance of a system to an external "applied force," which does not really exist, in that way, in Nature.

Second, the idea, that a "force" can act, without itself being changed or influenced by the system upon which it is acting. Newton's third law of action and reaction is not enough to remedy that flaw, because it assumes a simplistic form of point-to-point vectorial action, not existing in the real world.

The third, most essential fallacy lies in the attempt to break up the interactions of physical systems into a sum of supposedly elementary, point-to-point actions.

According to the author's standpoint—which he has assimilated from the study of Kepler and Leibniz—such "forces" as the gravitational pull the Earth appears to exert toward a rock, do not exist as isolated entities in the manner represented by Newtonian physics. "Forces" are merely effects derived from the unified, Keplerian physical geometry of the Universe. When we lift a rock from the Earth's surface, we are effectively doing work against the organization of the solar system as a whole, and not merely against a supposed, elementary gravitational force "emitted" by the Earth in isolation.

Similarly, the idea of an external force, while it may serve as a "useful fiction" (to quote an expression of Leibniz) for the treatment of certain problems in mechanics, should never be taken as more than that. An "external force" is a simplistic approximation, for what in reality is an interaction of physical systems—an interaction whose existence derives from the circumstance, that the interacting systems never existed as isolated entities in the first place, but only as subsystems of the Universe as a whole, as an organic totality.

These remarks, which could be elaborated much more, should help the reader to avoid falling into a number of confusions, which might otherwise arise from the paradoxical nature of Doubochinski's work. On the one hand, he employs tools of classical mechanics in his analysis of argumental oscillations; on the other hand, his entire approach, and the implications of amplitude quantization itself, imply a radical departure from concepts which have become almost self-evident in the academic teaching and practice of physics.

Historical Background

Danil Doubochinski emphasizes that argumental oscillations had already found wide application in the design of particle accelerators and electron tubes, as well as in investigations of so-called Fermi acceleration of cosmic rays, long before the Doubochinski brothers' original work in the late 1960s and 1970s.

Argumental oscillations had already appeared, around 1919, in the pioneering work of Barkhausen and Kurz on the generation of microwaves. They noted that oscillating electrons, interacting with the high frequency electromagnetic field in the tubes they had constructed, spontaneously organized themselves into "bunches," moving in equal phase with

Differential Equation for the Argumental Pendulum

The standard differential equation for a simple circular pendulum is

(1) $ml\ddot{\varphi} = -mg\sin\varphi$

where φ is the angular displacement of the pendulum from the vertical position and l is the length of the pendulum. The term $-mg\sin\phi$ represents the component of the force of gravity in the direction of the pendulum's motion. The equation is usually written:

$\ddot{\varphi} + \omega_0^2\sin\varphi = 0$, where $\omega_0 = (g/l)^{1/2}$

For small oscillations, $\sin\varphi \approx \varphi$ and the solutions of the corresponding equation $\ddot{\varphi} + \omega_0^2\sin\varphi = 0$, are simple sinusoidal oscillations $\varphi = a\sin(\omega_0 t + b)$, of frequency $f_0 = \omega_0/2\pi$ (called the proper frequency of the pendulum).

Equation (1) does not take into account the effect of frictional dissipation of energy; to do so, we must introduce a term $-\beta\dot{\varphi}$ on the right side of equation (1), where β is a coefficient expressing the effect of friction. This leads to the equation

(2) $\ddot{\varphi} + \beta\dot{\varphi} + \omega_0^2\sin\varphi = 0$

In the case of Doubochinski's pendulum, we have in addition an oscillating external force which acts only when the pendulum is inside the "interaction zone." The force can be expressed as $A\epsilon(\varphi)\sin(vt)$, where $\epsilon(\varphi) = 1$ for $|\varphi| \leq \varphi_0$, $\epsilon(\varphi) = 0$ for $|\varphi| > \varphi_0$. $F = v/2\pi$ is the frequency of the external field; A is its amplitude.

This leads to the full equation for Doubochinski's pendulum:

$\ddot{\varphi} + \beta\dot{\varphi} + \omega_0^2\sin\varphi = A\epsilon(\varphi)\sin(vt)$

In the case of small oscillations, when the pendulum remains inside the interaction zone, the equation reduces to:

$\ddot{\varphi} + \beta\dot{\varphi} + \omega_0\varphi = A\sin(vt)$

which is the classical equation for forced oscillations of a damped harmonic oscillator. (See box, p. 55.)

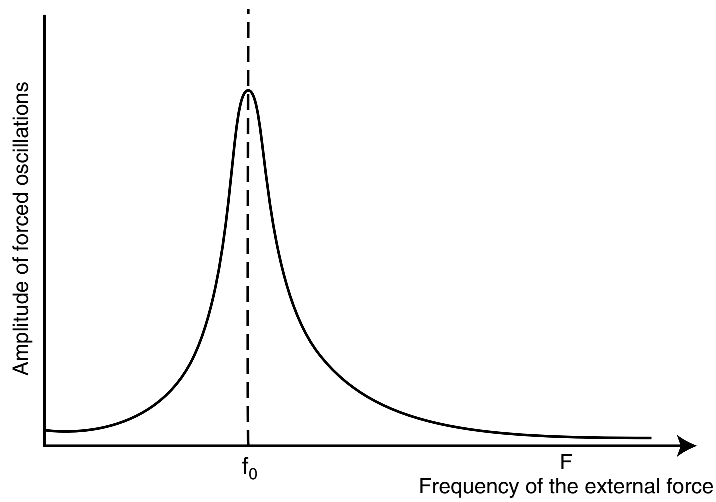
respect to the field. This "bunching effect" is crucial to the efficient transfer of energy from the electrons to the field, and has been widely exploited in the technology of high-power microwave generation until now, as well as in high-energy particle accelerators.[4]

The self-organization of an originally continuous stream of electrons into discrete "packets" is a reflection, on the microscopic scale, of essentially the same principle that causes the quantization of amplitudes in Doubochinski's pendulum. But until the work of Doubochinski and his collaborators, no one had demonstrated the corresponding phenomena of argumental oscillations in macroscopic systems on the laboratory scale, nor called attention to the universal nature and potentially revolutionary technological implications of these phenomena.[5]

The experimental realization and detailed investigation of argumental oscillations in macroscopic electromechanical systems, was originally carried out by Danil and Jakov Doubochinski and J.D. Penner at the Physics Department of the Vladimir Pedagogical Institute in the early 1970s. These investigations were continued at the renowned Lebedev Institute in Moscow, and at other locations in the Soviet Union. In addition to the Doubochinski pendulum, which we shall now examine in some detail, various other devices were constructed on the principle of argumental oscillations, including new types of electric motors having a discrete multiplicity of rotor speeds for one and the same frequency of the supplied current.

At the same time, it was realized that the phenomenon of amplitude quantification, although existing in reality, could not be demonstrated in standard large-scale computer simulations of the differential equations of motion. Specialized programs had to be developed *a posteriori*, to make certain aspects of argumental oscillations accessible to study with the aid of computers. Doubochinski has also developed mathematical methods for calculating the approximate values of the quantized amplitudes.

We now take a closer look at the pendulum, which provides the simplest and most striking experimental demonstration of amplitude quantization in argumental oscillations.



TYPICAL RESONANCE CURVE FOR FORCED OSCILLATIONS OF A DAMPED OSCILLATOR

A strong resonance interaction is produced with an external force of frequency F only when F is near f_0, the natural frequency of the oscillator.

The Two Regimes of the Doubochinski Pendulum

To get a first insight, into why the behavior of Doubochinski's pendulum differs so radically from that expected from textbook physics, it is useful to contrast two regimes of operation of the pendulum, presenting two very different physical-geometrical characteristics: First, the case of small amplitudes, where the pendulum remains entirely within the interaction zone of the electromagnet; and second, the case of larger amplitudes, in which the pendulum moves beyond the interaction zone.

The first case corresponds very nearly to the textbook case of "forced oscillations of a linear oscillator under a periodic external force" (See box, this page). Imagine that we release the pendulum from a position well within the interaction zone, not far from the vertical. The magnetic field being approximately uniform in that zone, the accelerating or decelerating action of the electromagnet is essentially independent of the pendulum's position. For small amplitudes, the pendulum behaves very much like an ideal linear oscillator, reacting to the "external force" of the electromagnet.

The standard textbook analysis tell us, that when the frequency of the current supplied to the electromagnet is large compared to the frequency of oscillation of the pendulum, the net effect of the alternating field on the pendulum's motion will be small, and the evolution of the pendulum's amplitude will not change significantly from what would happen, if the electromagnet were not present at all. This, in fact, is the behavior we actually observe. Evidently, the effects of rapidly alternating acceleration and deceleration tend to cancel out over any given period of the pendulum.

The behavior of the pendulum in this regime of small oscillations, conforms broadly to the standard textbook accounts. A significant transfer of energy from the alternating field to the pendulum's motion occurs only when the frequency of the electromagnet's field comes close to the natural frequency of the pendulum itself. This is the classical case of resonant oscillations. Notably, the amplitude of the pendulum increases with the amplitude of the external force—in this case, as a function of the voltage of the alternating current supplied to the electromagnet—and can take on an apparently continuous range of values. There is no quantization on the macroscopic scale.

Doubochinski remarks, that in this classical form of resonance, the oscillator appears to be rigidly "enslaved" to the external force.

The behavior of the pendulum becomes much more interesting, however, as soon as the pendulum has sufficient energy to move beyond the narrow zone of interaction. Leaving

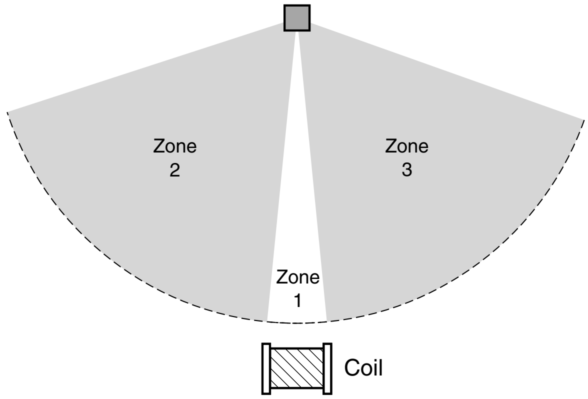


Figure 3
THE THREE ZONES OF MOVEMENT OF THE DOUBOCHINSKI PENDULUM
Zone 1 is the interaction zone; zones 2 and 3 are exterior zones.

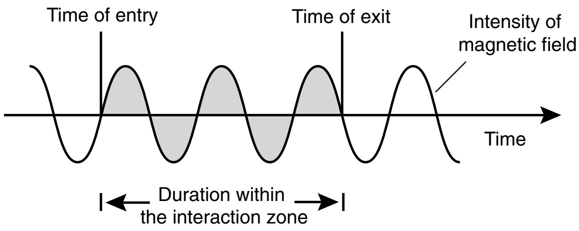


Figure 4
SINUSOIDAL FUNCTION OF THE MAGNETIC FIELD
Shown is the sinusoidal function of the magnetic field and included surface between the limits defined by the times of entry and exit from the interaction zone.

that zone, the action of the electromagnet on the pendulum drops off very rapidly toward zero, so the motion of the pendulum in the area outside the interaction zone is practically an undisturbed, free motion.

Thus, for large motions of the pendulum, we must distinguish among three different zones traversed by the pendulum (Figure 3): (1) the interaction zone and its immediate neighborhood, in which the alternating magnetic field exerts a significant influence on the pendulum; (2) the outer zone to the left of the electromagnet; and (3) the outer zone to the right of the electromagnet. In the latter two zones, the interaction between the magnetic field and the pendulum can be taken to be practically zero. The existence of these three zones implies—for the case of large motions—that the external force, acting upon the pendulum, is no longer independent of the pendulum's position, but depends on which of the zones the pendulum is located in at a given moment.

This circumstance fundamentally transforms the variety and character of the modes of exchange of energy between the pendulum and the alternating magnetic field. Most important, the process of "cancelling out" of alternating accelerations and decelerations of the pendulum by the alternating field in the interaction zone, is interrupted at the moment that the pendulum leaves the zone. If the alternating field completes a whole number of cycles during the time the pendulum traverses the zone, then the effects of positive and negative half-cycles will still cancel out; but if the number of cycles is not a whole number, then cancellation may not occur, and there can be a net transfer of energy between the pendulum and the field, during the former's passage through the interaction zone.

It is not hard to see, that the sign and absolute magnitude of the energy exchange, depend on the phases of the alternating field at the moment the pendulum enters and exits the interaction zone, relative to the direction of the pendulum's motion. For example, if the pendulum enters the interaction zone when the magnetic field is beginning a cycle, but leaves the zone in the middle of a succeeding cycle—that is, after an odd number of half-cycles—then there will be a non-zero, net transfer of energy (Figure 4). Assume the direction of motion of the pendulum, relative to the polarity of the field, is such that the initial half-cycle of the field has an accelerating effect. In this case, since the total number of decelerating half-cycles will be one less than the number of accelerating half-cycles, after cancellation of pairs of oppositely acting half-cycles, the net effect will be equivalent to that of the first half cycle.[6] In this case, the pendulum will gain energy. If the pendulum enters the field at the same phase of the current, but in the opposite direction of motion, the net effect will be a deceleration and a loss of energy.

Now, note that the exact relationship between entry and exit times depends not only on the entry velocity and direction, but also to a significant extent on the phase of entry. This is because the total "time of flight" through the interaction zone is modified by the changes in velocity caused by the alternating field. As a result, the amount of energy gain or loss during a single passage through the zone, is a complicated function of the entry velocity and entry phase.

Careful investigation shows that for a given entry velocity there always exist phases of entry, for which the pendulum enjoys a net increase in energy, as well as phases for which a net decrease occurs. The possibility of a net energy gain means that the pendulum—provided it can somehow "choose" the right phases of entry into the interaction zone—might be able to draw exactly as much power from the alternating field as it needs to overcome its frictional losses, and thereby maintain itself in a stable regime of motion.

The overall behavior of the pendulum will depend, however, on the cumulative effect of many successive passages through the interaction zone. The phases of entry into the zone can change from one entry to the next. Any attempt to foresee what will happen, from an *a priori,* mechanistic standpoint, leads us into a seemingly endless labyrinth of complexities, typical for what engineers and physicists broadly term "nonlinear problems." Given the lack of adequate, general theoretical principles for handling such problems, industrial engineering practice is obliged to resort, in each specific case, to an *ad hoc* combination of mathematical studies, computer simulations, and experiments, often expending large resources for this purpose.

Also in this case, the only reliable approach, at the outset, is to actually build the pendulum and see what it does! A combination of hypothesis, experiment, and theoretical analysis allowed Doubochinski and his collaborators to identify certain crucial features of the process, connected with the emergence of a discrete, "quantized" array of stable amplitudes in the pendulum. These, in fact, apply to a much broader class of oscillating systems, subsumed under a common principle of "argumental interaction."

At the risk of taxing the reader's attention somewhat, we propose in the following paragraphs to go into the mentioned, crucial features of argumental oscillations in some depth. Full details of experiments and mathematical analyses are available in a large number of scientific publications from the Soviet period, most of which, however, are available only in the Russian-language original.

Velocity Modulation

The first observation we take up here, played a key role in the genesis of the Doubochinski brothers' original discovery.

If we assume, at the outset, that the period of the pendulum (at a given amplitude) is incommensurable with the period of the alternating current supplied to the electromagnet, then we should expect the phases of the field, at the moment the pendulum enters the interaction zone, to be randomly—that is uniformly—distributed among all possible values between 0 and 360 degrees. One might conclude, in that case, that the net result of the interaction, over many periods of the pendulum, would be essentially zero. Indeed, for each given phase of entry, the opposite polarity of the field (relative to the pendulum's direction of motion) would occur equally often, and, since the forces exerted on the pendulum, as a function of time, are exactly opposite for the opposite relative polarities, their effects would cancel out, on the average.

This reasoning, however, overlooks the possible effect, already mentioned above, of changes in the pendulum's net velocity, and thereby also in the time during which the pendulum remains in the interaction zone, as a result of the interaction with the electromagnet. As it turns out, that effect introduces a surprising asymmetry into the process, leading to a situation, in which the pendulum can draw a net positive power input from the electromagnet, even without a tight correlation of phase having been established.

The principle involved is illustrated by Figure 5. Here the sinusoidal curve represents the accelerating or decelerating force of the field generated by the electromagnet, relative to the motion of the pendulum; and the vertical line at left and the arrows at right plot the moments of entry and exit of the pendulum from the "interaction zone." Evidently, the net change of velocity of the pendulum, between entering and exiting the zone, will be equal to the integral of the accelerating/decelerating force acting on it in that zone; that is, the total area bounded by the curve between the entry and exit times, with the portions under the *x*-axis counted negatively.

Designate the width of the interaction zone by d, the velocity of the pendulum at the point it enters the interaction zone by v_0, and the period of oscillation of the current supplying the electromagnet by T. Thus, if the field were turned off, the pendulum would traverse the zone in a time t_0 equal to d/v_0. Assume that the field of the electromagnet is not too strong, so that the change in velocity of the pendulum, as a result of a single passage through the field of the electromagnetic interaction zone, is only a small fraction of its velocity v_0. This is the normal operating situation of Doubochinski's pendulum.

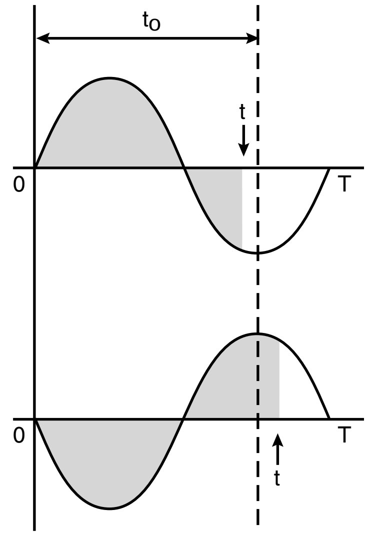


Figure 5
THE EFFECT OF VELOCITY MODULATION FOR OPPOSITE PHASES
Shown are the effects of velocity modulation for one pair of opposite phases. The arrows mark the moments of exit of the pendulum from the zone of interaction.

Now, consider different possible relationships between the "transit time" t and the period of the electromagnet. Evidently, if t is exactly equal to a whole period T of the electromagnet, then the effect of the positive and negative phases of the field would cancel out, and the pendulum would exit the interaction zone with the same velocity v_0, as when it entered.

For a larger velocity v_0 (larger amplitude), we will have $t < T$. Assume, for the sake of illustration, that $t_0 = 3/4T$. This is the situation presented in Figure 5. In the first diagram (a) we consider the case, where the pendulum enters the interaction zone at the very beginning of an accelerating phase of the electromagnet. Because the pendulum experiences a whole accelerating phase, but only part of a decelerating phase, the net effect will obviously be an acceleration of the pendulum. Compare this with the effect of the opposite phase (b), where the pendulum experiences a net deceleration. Now, if the time of interaction were the same in both cases, then the two effects would be equal and opposite. Observe, however, that in the first case (a), because of the net acceleration of the pendulum's velocity, the "transit time" t of the pendulum will no longer be equal to t_0, but will be slightly shorter, with a decelerating effect.

Similarly, in the second case, the net deceleration of the pendulum will cause it to stay in the interaction zone slightly longer than t_0. The result of this, as the reader can easily see, is that the areas under the two curves, from the moment of entry to the moment of exit from the zone, are no longer equal and opposite: The net accelerating effect (that is, net increase in the kinetic energy of the pendulum) in case (a) will be larger than the net decelerating effect in case (b). Taken together, the two would result in a net gain in energy by the pendulum.

What happens when the pendulum enters the interaction zone at a different phase of the alternating current than at the beginning of a cycle? A thorough analysis of all possible cases—which for brevity we omit here—shows that although some phases yield a net acceleration, and others a net deceleration, the effect of velocity modulation is to produce an overall net gain in energy of the pendulum, when averaged over many randomly distributed phases.

An early paper, published in *Soviet Physics Uspekhi* in 1973, summarized the situation as follows:

> The assumption that oscillations maintained by a harmonic force always assume the frequency of this force or a multiple of this frequency is widely used in the theory and practice of mechanical oscillations. However, inertial, thermal, and other effects, frequently not taken into account, introduce time shifts between the driving force and the dynamic functions of the oscillations, and can lead to an asynchronous excitation of undamped oscillations. . . . When the frequency of the oscillating system is not commensurate with that of the alternating external force, a resultant positive energy contribution (averaged over a number of oscillations) is possible if the system alters the time of flight through the interaction zone sufficiently strongly.

In particular, these results imply the possibility for the oscillating system (the pendulum in our case), of drawing power from a much higher-frequency alternating field, to compensate its frictional losses, and even increase its amplitude in the course of many oscillations.

Estimates of the net average energy transfer to the system, showed that it can take alternately negative as well as positive values, depending on the relationship between the nominal transit time t_0 and the period of the alternating field T.

If, for example, t_0 is equal to $\frac{1}{4}T$, rather than $\frac{3}{4}T$, then one can easily convince oneself that the average effect will be net loss of energy by the pendulum, rather than a net gain. Since complete cycles of the field have no net effect on the velocity of the pendulum, the values $t_0 = \frac{1}{4}T + T, \frac{1}{4}T + 2T, \frac{1}{4}T + 3T,$ and so on, and $t_0 = \frac{3}{4}T + T, \frac{3}{4}T + 2T, \frac{3}{4}T + 3T,$ and so on, will yield the same sign of effect—that is, a net overall deceleration or net overall acceleration—as $t_0 = \frac{1}{4}T$ and $t_0 = \frac{3}{4}T$ respectively.

This difference in behavior, depending on the nominal transit time of the pendulum through the interaction zone—that is, on the velocity v_0, which in turn is a function of the pendulum's amplitude—already points to a potential mechanism for the selection or quantization of amplitudes: Amplitudes for which losses dominate, will be damped out, whereas those that can draw net energy from the source, can sustain themselves.

Phase Synchronization

However, detailed experimental studies of the Doubochinski pendulum, revealed a second, crucial phenomenon involved in the emergence of precisely determined, dis-

How to Build a Doubochinski Pendulum

The construction is very simple (see Figure 1). The oscillating mass consists of a short permanent magnet, with one of its poles pointing downward, attached to the end of a wooden rod (or other rigid arm) 30 to 60 cm in length. When crossing the position of equilibrium, the pendulum crosses over a flat-shaped electromagnet (solenoid) 9 to 12 mm in width, whose axis is parallel to the pendulum's motion. The solenoid is supplied with alternating current from the household net, through a resistive load or from a transformer.

The windings of the solenoid should run perpendicular to the pendulum's plane of oscillation. The solenoid itself is mounted symmetrically, relative to the equilibrium (the lowest) position of the pendulum.

For the best results, observe the following details in the construction of the argumental pendulum:

The form of the permanent magnet at the end of the pendulum should be square or rectangular (a flat shape is best). One can also use a magnet of irregular form, for example a fragment of dimensions approximately 8 × 10 × 10 mm, broken off from the ferrite magnet of a loudspeaker).

As stated above, the magnet should be mounted on the end of the pendulum arm in such a way, that one of its poles points vertically downward toward the solenoid when crossing over it. In fact, the effect can also be obtained with a horizontally positioned magnet; however, in this case, the values of the discrete amplitudes will be somewhat different from those corresponding to the vertical orientation. The permanent magnet can be directly glued to the rod or fastened to it with the help of strong adhesive tape. A suitable pendulum arm, with a cross-section of 3 × 5 mm, can be fashioned from the material of a wooden ruler.

The suspension of the pendulum must have low friction. This requirement can be fulfilled quite well by using suspension mechanisms taken from discarded electrical measurement devices, of the sort that have moving pointers, such as voltmeters, multimeters, and so on. Generally, any suspension can be used that permits motion in a fixed plane with very little damping of the oscillations. Fix the pendulum arm to the suspension using a thin brass or aluminum fastener.

The solenoid can be a rectangular coil of dimensions 10 × 30 × 100 mm, wound with insulated wire of diameter 0.15 mm. The number of windings in the coil should be about 800. The optimal voltage for routine operation of the pendulum, is around 70 V. However, in order to test the effect of differing field strengths on the behavior of the pendulum, one can introduce a rheostat into the power supply, allowing the voltage to be varied continuously from 10 to 200 V. In the higher voltage range, it may be necessary to keep the time of operation short, to avoid overheating of the coil.

This is based on an article by D.I. Penner, M.I. Korsakov, Danil Doubochinski, and Yakov Doubochinski, published in the Soviet journal Physics in School, *1981.*

crete amplitudes and the remarkable stability of the quantized modes: After the release of the pendulum from a given position, the phases of entry into the interaction zone soon cease to be distributed in a random manner. The pendulum adjusts its motion in such a way, that its entry into the zone becomes very nearly synchronized with a specific phase of the alternating field. This "auto-synchronization" of the pendulum is analogous to the effect of "phase bunching" of electrons or other charged particles in a high-frequency electromagnetic field, as mentioned earlier. Careful analysis shows that the auto-synchronization tendency of the pendulum is itself closely connected with the mechanism of velocity modulation, which we just examined. Let us now see, how that synchronization tendency in turn leads to a discrete series of amplitudes for the pendulum.

Observe, first, that the pendulum passes through the interaction zone twice for each full period—once in each direction. Between any two successive passages, the pendulum swings freely in the outside area, up to a certain maximum height, and then swings back to enter the interaction zone in the opposite direction. That process takes a certain time, between the moment the pendulum exits the interaction zone and the moment it reenters that zone. Consequently, the phase of the alternating field at the moment of each new entry into the interaction zone, depends on the phase at the moment of the preceding exit from that zone, and the time between those two moments.

Next, take account of the important fact, that the elapsed time between successive exit and reentry into the interaction zone—and thus the relationship between the phases of successive entries into that zone—depends on the amplitude of the pendulum's motion. For larger amplitudes, the pendulum takes slightly longer to arrive from its maximum height to the interaction zone near the bottom of its swing. This fact is related to a property of a circular pendulum, which should be known to any student of classical physics and is commonly referred to as anisochronicity: The period of oscillation is not fixed, but depends on the amplitude of the pendulum's motion.[7]

The dependence of periodic time on the amplitude, opens up a new possibility for our system, which is entirely absent in the classical linear systems: namely, to use its variable amplitude as a means for regulating its phase relationships with respect to the alternating field.

It is also important to note, by the way, that even a nominally linear oscillator, when subjected to the velocity-modulating influence of a spatially inhomogeneous alternating field, can take on anisochronic characteristics that permit it, too, to regulate its phase relationships in similar manner to the Doubochinski pendulum.

Returning to the pendulum, let us suppose, to be concrete, that the alternating field has a frequency F = 50 Hz, and the pendulum's characteristic frequency (the frequency for very small amplitudes) is 0.5 Hz (a period of 2 seconds).

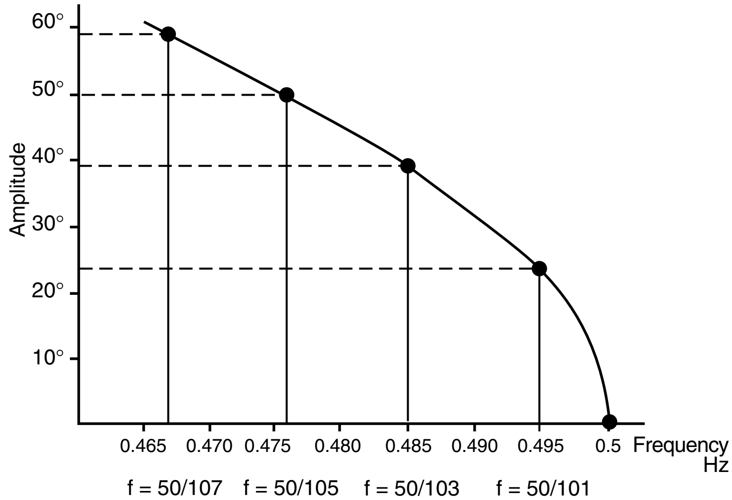


Figure 6
RELATIONSHIP OF AMPLITUDE TO FREQUENCY
The relationship of amplitude to frequency in a circular pendulum is shown. The approximate formula is $a \approx 162 \sqrt{(1 - (f/f_0)^2)}$*, where f is the frequency and* f_0 *is the natural frequency (small oscillations), which, in our example, is equal to 0.5 Hz.*

Figure 6 illustrates the dependency of the pendulum's momentary frequency on its amplitude, for a typical choice of physical parameters of the pendulum. Note, that for certain values of the amplitude, the frequency of the alternating field (50 Hz) will be an integral multiple of the frequency of the pendulum—that is, when the alternating field performs a whole number of oscillations during a single period of the pendulum. As a result, the phases of the alternating field, at which the pendulum enters and exits the interaction zone, will be repeated from one cycle of the pendulum's motion to the next, opening up the possibility of a stable, "stationary" regime.

The first case of this arises for "infinitesimally small" pendulum motions, whose frequency is f = 0.5 Hz. The ratio of frequencies is F/f = 50/0.5 = 100. In this case, however, the pendulum remains inside the interaction zone and behaves essentially as predicted by the classical theory of resonance: The frequency of the "external force" being many times larger than the characteristic frequency of the pendulum, there is practically no effect on pendulum's average motion, and there is no quantization of the amplitude.

For larger amplitudes, the period of oscillation of the pendulum will be slightly longer, and its frequency lower, leading to a larger value for the ratio F/f. The next whole-number value, larger than the value 100, would be F/f = 101, which would occur for a pendulum frequency of f = 50/101 Hz = approximately 0.495 Hz. Looking at Figure 6, we see that this corresponds to an amplitude, in terms of maximum angle of deflection from the vertical, of about 23 degrees.

In this case, the pendulum's motion goes beyond the bounds of the interaction zone; for each full period, it passes twice through—once in each direction. The time between the

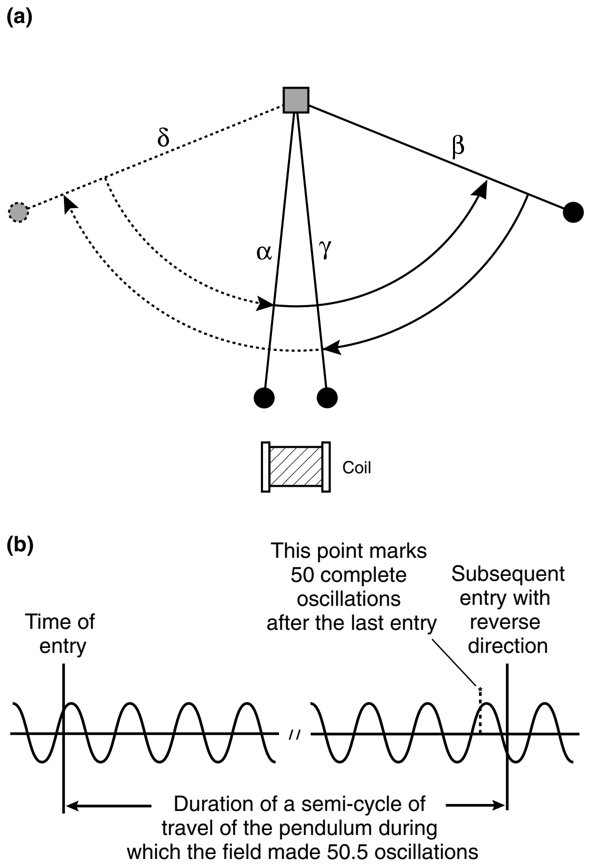


Figure 7
PHASE RELATIONS

Shown in (a) are half-cycle movements of the pendulum during two successive journeys through the interaction zone. The pendulum enters the zone at α, traverses the zone, reaches a maximum height at β, and enters the interaction zone in the inverse direction γ. During the next half-cycle, the pendulum goes from γ to δ, and from δ to α.

The oscillations of the sinusoidal function of the magnetic field during the two successive entries of the pendulum into the interaction zone are shown in (b), where one sees that the second entry is made with reverse phase (displaced by 90° in relation to the first entry).

moment of entry in one direction, and the next moment of entry in the opposite direction, corresponds to a half-period of the pendulum, which in turn corresponds to 101/2 = 50.5 oscillations of the alternating field (Figure 7). Thus, in the time between two successive passes through the interaction zone, the alternating field makes exactly a whole number of cycles plus a half-cycle. This means that the pendulum, when reentering the interaction zone after a given passage, will encounter the alternating field in a phase which is shifted by 180 degrees—that is, the opposite phase—relative to that of the previous passage. Since it is also moving in the opposite direction, the effect on the pendulum will be to accelerate (or decelerate) it by exactly the same amount as in the previous passage. In other words, the gain (or loss) in energy of the pendulum will be the same for both of the two successive passes through the interaction zone.

Ratio F/f	101	103	105	107	109	111
Observed amplitude	30°	43.2°	53.2°	59.9°	68°	74.2°
Calculated amplitude	22.8°	39.1°	50°	58.6°	65.9°	72.1°

Table 2
OBSERVED AND CALCULATED AMPLITUDES

Now, as we noted earlier, there will always exist phases of entry into the zone, for which the pendulum receives a net surplus (or net loss) of energy. If the friction in the pendulum is not too large, then a phase will exist for which the gain in a single passage through the interaction zone, exactly balances the frictional loss in a half-cycle of the pendulum. If we release the pendulum at the right amplitude (the approximately 23 degrees we determined above) and at the right moment (so it enters the interaction zone at the appropriate phase), then it will reenter the zone after a half-period in the opposite direction and in exactly the opposite phase, pick up the exact same energy gain, and reenter the zone once again in the correct phase after another half-cycle. We will thus have a so-called "stationary regime" in which the pendulum maintains a constant amplitude, drawing just as much power as it needs to overcome its frictional losses.

Several remarks are important to make at this point.

First, our discussion actually points to the potential existence not only of one, but of a discrete series of stationary regimes. The essential parameter is the ratio of frequency of the alternating field to the frequency of the pendulum. In our discussion above, we saw that a stationary regime is possible for $F/f = 101$. It is easy to see, however, that the same argument applies whenever the ratio F/f is equal to an odd whole number, that is, 103, 105, 107, and so on. For our chosen case of $F = 50$ Hz, these values correspond to $f = 50/103$ Hz = 0.485 Hz; $f = 50/105$ Hz = 0.476 Hz, $f = 50/107$ Hz = 0.467 Hz, and so on. Looking at Figure 6, we can read off the values of the pendulum's amplitude, which correspond to these frequencies.

Second, we did not take account, above, of the slight changes in the momentary relationship between amplitude and frequency of the pendulum, caused by the interaction with the electromagnet over a single half-cycle.

For these and other reasons, although our analysis strongly suggests the existence of "stationary regimes," it by no means proves that they can actually be realized in practice. For example, how does the pendulum "find" the suitable phases and amplitudes? In experiments, the pendulum actually demonstrates its ability to do this, but a truly comprehensive theoretical explanation has not been given.

Self-Regulation

In fact, the theoretical amplitudes, calculated above, do display a rough correspondence to the "quantized amplitudes" actually observed in experimental realizations of

Doubochinski's pendulum. The discrepancies, caused mainly by the effects of friction and the changes in velocity inside the zone of interaction, are largest for the smallest amplitude observed (typically 30 degrees, as opposed to the calculated 23 degrees).

On the other hand, the actual quantized motions realized in Doubochinski's pendulum do not correspond exactly to the ideal stationary motions described above, but are much more complicated. They agree only in average with the idealized motions. What occurs, in first approximation, is that the actual phases of entry into the interaction zone "wander" around the values corresponding to "pure" stationary motions.

These experimentally observed phenomena are best described in terms of the so-called "phase-space diagram" (Figure 8). When the system is disturbed, its phase-space trajectory begins to "orbit" around the motion corresponding to the stationary regime. If the disturbance is not too large, this "orbit" is gradually dampened out, and the system's phase-space trajectory is a spiral, converging toward a small "wandering" motion in the vicinity of the stationary regime. A major disturbance, however, can throw the pendulum into a completely different phase-space region. In some cases, the pendulum executes a "quantum jump" to a different quantized amplitude.

In fact, depending upon the initial conditions, even much more complex behavior is possible—for example a motion, which spontaneously jumps back and forth between quantized amplitudes, imitating the behavior of some atomic systems in quantum physics. Nevertheless, the stable, "quantized" amplitudes prevail as the "favorite" modes of Doubochinski's pendulum, and are by far the easiest to demonstrate.

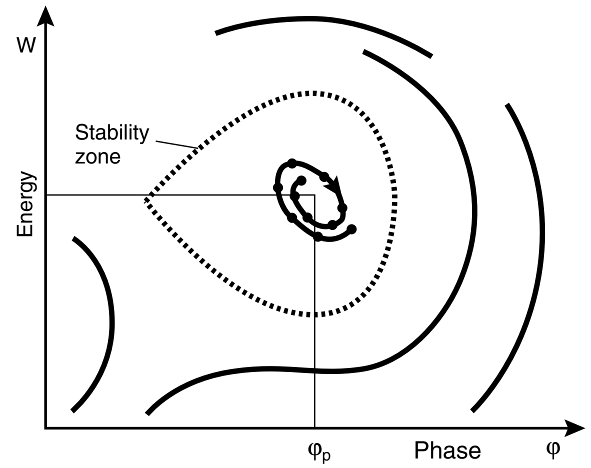


Figure 8
PHASE DIAGRAM OF PENDULUM TRAJECTORY
In this phase diagram, φ_p is the phase corresponding to a "stationary system."

Stability Properties

All of this would be little more than a scientific curiosity, of little practical import, were it not for the fact that the quantized regimes of the Doubochinski pendulum and other, suitably constructed argumental oscillators, display an extraordinary stability relative to both large variations in the strength of the driving force (that is, the current in the electromagnet), and to external perturbations of various kinds. Both these characteristics are crucial to the technological applications of argumental oscillations.

In experiments carried out by the Doubochinskis and their collaborators on the argumental pendulum, the voltage supplied to the electromagnet was varied over a wide range, starting with the lower limit at which the stable quantized oscillations appeared, up to a value nearly 20 times higher. The frequencies of the quantized regimes remained strictly constant, and amplitudes varied by less than 1 percent.

Precise observations revealed the mechanism of this remarkable adaptability, which holds true for argumental oscillations in general: It is by shifting the phase of entry into the interaction zone, while keeping the frequency and amplitude relatively constant, that the pendulum is able to maintain its "regime," compensating for changes in the strength of the source by modifying its interaction with the electromagnet. The same apparent "phase intelligence" of the pendulum, permits it to defend its quantized regimes—within certain limits, of course—against external disturbances of various kinds.

A general characteristic of argumental interactions between a high-frequency "source" and a low-frequency oscillator, is that the oscillator adapts to the "source" mainly by variations of phase, while its overall period of oscillation remains close to that corresponding to its own "proper frequency."

In this context, as demonstrated in countless experiments with a wide variety of mechanical, electromechanical, and electronic devices, the span of frequencies that can be efficiently coupled to each other in this way, can be enormous: For example, Doubochinski's pendulum, whose "natural" frequency is in the range of 0.5 Hz, can maintain itself in stable oscillations at nearly the same frequency, by drawing energy from a source (the electromagnet) operating at 1,000 Hz or more. With other devices, the frequency ratio can be even very much larger. As noted above, the spectrum of stable regimes is a function of the frequency applied to the system, and becomes richer and denser, the greater the ratio between the external frequency and the "natural" frequency of the oscillator.

These features open up the possibility of "feeding" a large number of oscillating systems, each at its own frequency, different from the others, by a single high-frequency source.

Finally, we should note, that the theoretical analysis of the stable motions, sketched above, depends quite essentially on two assumed properties of the oscillator (that is, the pendulum): (a) its anisochronicity, which allows the system to satisfy the condition F/f = an odd whole number, by adjusting the value of f; and (b) the existence of frictional dissipation, which appears essential to the stability of the quasi-stationary regimes. In reality, however, careful experiments have shown that the quantization phenomenon occurs even in the absence of these assumptions—a fact of physics not accounted for by the mathematical models. Once again we are confronted with evidence of a new physical principle.

Argumental Oscillations and Planck's Quantum of Action

As we noted earlier in this article, Doubochinski's pendulum is only a convenient pedagogical example of a very large

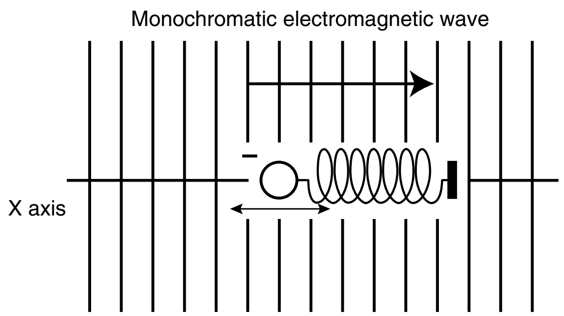


Figure 9
DIAGRAM OF THE DOUBOCHINSKI-PLANCK OSCILLATOR
The differential equation according to classical physics is $\ddot{x} + \beta\dot{x} + \omega_0^2 x = A \sin(vt - kx)$. *$v = 2\pi F$; k = wave number.*

class of oscillating systems, in which the strength of the "external force" depends on the momentary position (or configuration) of the system, and not only upon the time. More difficult to realize in a simple mechanical model, but more natural from a physical standpoint, is the case of a spatially extended oscillator—idealized here as a charged body fixed to a spring—interacting with a high-frequency electromagnetic field (Figure 9).

With one very important difference, this case closely resembles the picture of the "elementary oscillators" considered by Max Planck in his studies on so-called "blackbody radiation." Blackbody radiation was conceived hypothetically as the equilibrium radiation field resulting from the emission and absorption of electromagnetic radiation by a large number of atoms or molecules in a cavity with reflecting walls. Regarding the atoms in first approximation as an aggregate of "elementary electromagnetic oscillators," Planck counterposed the hypothetical spectral distribution of blackbody radiation as a function of temperature, predicted by calculations based on the commonly accepted, Maxwellian "laws of electrodynamics," to the completely different spectral characteristics actually observed in experiments. To account for this gross discrepancy, Planck hypothesized a new physical principle, referred to as the "elementary quantum of action," shaping the interaction between the oscillators and the radiation field in a manner which contradicts the assumptions of Newtonian-Maxwellian physics. The universal character of Planck's quantum hypothesis was subsequently confirmed in countless experiments.

However, the hypothetical "elementary oscillator," which Planck chose as the starting-point for his original analysis, was essentially equivalent to the one assumed in the classical case of "forced oscillations." In particular, the spatial extension of the oscillator itself is ignored in characterizing its interaction with the radiation field.

What happens if we drop this arbitrary assumption, and consider instead the case, where the amplitude of the oscillating body's motion is not small relative to the wavelength of the field? In that case, as it moves, the body experiences the field at different positions, as well as times. We thus have a case of "argumental oscillations" of a somewhat different sort than Doubochinski's pendulum; but which, in agreement with Doubochinski's principle, turns out also to have a discrete set of "quantized" amplitudes.[8] Those values can be calculated on the basis of general methods that he and his colleagues have developed.

The interesting point is, that Doubochinski's analysis does not depend in any explicit way on Planck's quantum of action, nor does it presuppose that the system be microscopic in scale. Laboratory experiments, carried out by Doubochinski and his collaborators on macroscopic systems simulating the idealized system under consideration, exhibit quantized amplitudes whose values are close to those predicted by his methods. This suggests that Doubochinski's discovery reflects a still more comprehensive principle than the Planck quantum of action, as presently understood—a general principle that would embrace the microscopic quantum of action, macroscopic quantization as demonstrated by Doubochinski's pendulum, and the characteristic quantization of astronomical systems, including not only the planetary orbits, but also such things as spiral-galactic arms and the rates of rotation of many astronomical objects. In fact, such a general principle is already implicit in the work of Johannes Kepler. It opens up a vast domain for further research.

It is important to emphasize the fundamental difference between Danil Doubochinski's approach and that of various physicists and mathematicians who, over the years, have attempted to derive the "quantization" of microscopic systems from classical mechanics, by introducing "nonlinear" terms in a more or less arbitrary way into the equations of motion. As we explained above, Doubochinski's amplitude quantization is an experimentally confirmed discovery of a real physical effect, which cannot be mathematically deduced from classical mechanics. Also, Doubochinski does not attempt to deduce Planck's blackbody radiation law or the laws of quantum mechanics from his principle; he merely calls attention to the striking coherence between quantum phenomena on the microscopic scale, and the behavior of argumental oscillations on the macroscopic scale.

Mathematical Methods

The essence of Doubochinski's general method for calculating the values of the quantized amplitudes is worth briefly mentioning here, because it provides a more synthetic viewpoint on the phenomena which we examined above, in somewhat painful detail, in the case of the pendulum.

We assume, in agreement with experiment, that the oscillating system, moving under the influence of a spatially inhomogeneous, high-frequency "field," will execute a quasi-periodic motion whose basic period is close to that of its natural, undisturbed motion, but whose amplitude and phase may vary in a certain fashion in adapting to the "field." Any periodic motion through a spatially inhomogeneous field has the effect of "modulating" the time-function of the external force, experienced by the oscillator, in such a way as to generate a large array of harmonic frequencies "spaced" at integral multiples of the base-frequency of the oscillator itself (Figure 10). If the external frequency is equal to an integral

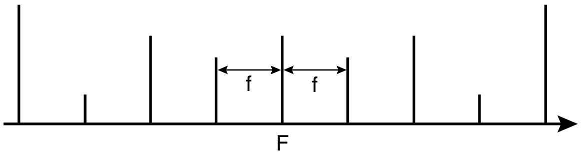


Figure 10
SPECTRUM OF SELF-GENERATED HARMONICS
Shown is the region of self-generated harmonics by the movement of the pendulum in the function that determines the action of the external force. The harmonics are frequencies of the approximate form of F – nf, where n is a whole number.

multiple of the undisturbed characteristic frequency of the oscillator itself, then that "natural" frequency will be among the harmonics. As a result, the pendulum can bring itself into resonance with the "signal," generated by its own space-time modulation of the external field, thereby drawing the power needed to maintain a stationary regime!

Those interested can find a detailed presentation of this method in the technical literature.[9] Here I only note the following: In the case of Doubochinski's modified Planck oscillator interacting with a monochromatic electromagnetic wave, the mathematical expression for the harmonics is connected with the trigonometric series for the result of the frequency-modulation of a sine wave by another sine wave of different frequency, which is well-known from radio technology. The coefficients of that series are given by Bessel's functions. In this manner, Doubochinski arrives at a formula for the quantized amplitudes of the oscillator in terms of extrema of the Bessel functions. In the case of Doubochinski's pendulum, one obtains very nearly the same values, as derived from our more elementary consideration of stationary regimes.

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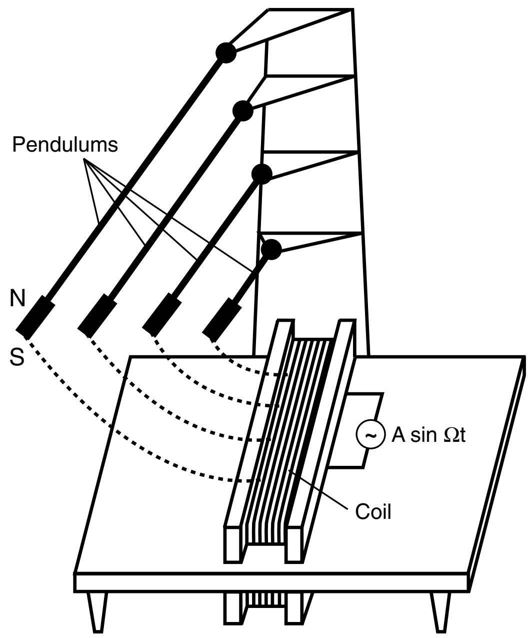

Figure 11
MULTIPLE ARGUMENTAL PENDULUMS
Multiple pendulums with different natural frequencies are driven by a single high-frequency magnetic field.

Notes

1. The photoelectric effect concerns the emission of electrons from a metal when irradiated by light. It is found that the energy of the individual emitted electrons is largely independent of the strength (amplitude) of the light, but increases with its frequency. For an atom or molecule irradiated by light, the energy levels to which the atom can be excited depend on the frequency of the light, but—except for extremely intense light—not on the light intensity. Generally speaking, the higher the frequency, the larger the range of discrete states that can be excited, up to the point of ionization.
2. By their very nature, standard computer algorithms for the solution of differential equations introduce artifacts which are present neither in the actual mathematical function described by the equation, nor in the real physical process. In the present case, where the value of the quantized amplitude is necessarily a discontinuous function of the initial conditions, the commonplace algorithms are doomed to fail. To develop useful computer methods for this sort of problem, it is necessary to take account of the essential features of the physical process, as demonstrated by actual experiments.
3. Doubochinski compares the mean radii of the planetary orbits with the calculated series of quantized amplitudes of a simple argumental oscillator (essentially Doubochinski's version of the Planck oscillator, described in the present article), scaled to the Earth orbit radius as "1." He finds that the values for the orbital radii agree quite closely with quantized amplitudes of the argumental oscillator (see Note 9). The latter series contains many amplitudes which do not correspond to observed planetary orbits; these amplitudes would correspond, if the analogy with a simple argumental oscillator holds up, to possible orbits which are not occupied in the present solar system.

   These additional orbits are not permitted, however, by Kepler's harmonic laws. The latter, it should be noted, identify key features of the solar system— particularly the unstable region of the asteroid belt—which are not accounted for by Doubochinski's simple model, and point to the action of a higher principle. This being said, Doubochinski's preliminary results are of great interest, pointing in the direction of an oscillatory theory of gravitation, for which many indications already exist.
4. Doubochinski points out that the functioning of the original Hertz oscillator, with which Heinrich Hertz first demonstrated the transmission of electromagnetic waves in 1888, depends on an effect of nonlinear "bundling" of electrons in the electrical discharge exciting the oscillator, which was not known or understood in Hertz's time.
5. In the 1940s and 1950s, Rocard observed the existence of stable regimes in a pendulum interacting with an oscillating magnetic field, and also wrote down a differential equation to describe the motion. However, he was not able to arrive at a satisfactory understanding of the phenomenon, nor did he apparently observe the quantization of amplitudes.
6. According to classical mechanics, the net change in velocity in traversing the zone of interaction, is equal to the time-integral of the force acting during the corresponding time, that is, the total area under the sine-wave curve enclosed between the moments of entry and exit from the zone.
7. Interestingly, Doubochinski's pendulum restores the "lost isochronicity" of the circular pendulum, by evolving toward a stable regime in which a constant amplitude is maintained.
8. It is worth noting, that in this case the dependence of the external force on the position of the oscillating body is a continuous function. Nevertheless, amplitude quantization occurs, just as in the pendulum, but with a different discrete series of amplitude values.
9. See, for example, the paper by D.B. and J.B. Doubochinski, "Amorçage argumentaire d'oscillations entretenues avec une série discrète d'amplitudes stables," *EDF Bulletin de la Direction des Etudes et Recherches,* Série C, Mathématiques, Informatique, No. 3, 1991, pp 11-20.